Classic and Modern Measures of Risk in Fixed Income Portfolio Optimization

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Purpose - The interest rate risk immunization is one of the key concerns for fixed income portfolio management. In the last years, the affluence of new risk measures emphasizes the importance of comparing them to the classic approaches. As a result, one question arises: Which is the relation among classic risk measures (Macaulay Duration, Convexity and Dispersion…) and other more recent risk measures (such as Value at Risk and Conditional Value at Risk) as tools for the formation of an optimum investment portfolio?

Design/methodology/approach - In order to obtain objectivity, an empirical study has been conducted on the US Treasury bonds market by means of the formation of different portfolios among a selected set of bonds with different maturities and structures. In addition, information about yields from middle 1990's and early 2000's has been used to find the optimum portfolio compositions based on each alternative risk measure.

Findings – The main finding of the study is that there is an absence of relationships between those portfolios optimized by classic measures and those optimized by modern measures. The results show how both types of risk measures lead to quite different portfolios.

Practical implications – The behavior of modern risk measures has been examined, finding that when VaR is used, the sensitivity of the optimal portfolio with respect to the level of confidence is too high. Finally, if CVaR is used, then the optimal portfolio is quite stable with respect to the confidence level.

Original/value – This is the first paper that compares classic and modern measures of interest rate risk in fixed income portfolios. It is of value to decision makers, experts and economic researchers.

Keywords - Immunization risk measures; dispersion measures; maturity-matching bonds; value-at-risk (VaR); conditional value-at-risk (CVaR); expected shortfall; coherent risk measures; portfolio optimization; risk management

Paper Type - Research paper

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INTRODUCTION

Duration and Convexity are very well known concepts in the area of interest rate risk management. The simplicity that characterizes both measures arises from the assumption that the movements of the term structure of interest rates are parallel. In that line, the problem of simple immunization was introduced by Fischer and Weil (1971). Once the unreal assumption of parallel movements in term structure of interest rates is abandoned, new and improved sensitivity measures arise, such as dispersion measures. Following a chronological sequence, the more distinctive dispersion measures are the M^2 of Fong and Vasicek (1984) and the $\tilde{N}$ of Balbás and Ibáñez (1998).

These classic measures of interest rate risk do not consider the real behavior of the interest rates by means of historical or simulated data. This imposes a limitation in the portfolio optimization that modern measures try to solve. The Value-at-Risk (VaR) is a popular risk measuring methodology, which has been used as the base for the industry of regulation, see Jorion (1996), Pritsker (1997) and Szegö (2002). Nevertheless, VaR has been seriously criticized when returns are not 'normally' distributed (for instance, when profit and loss distributions tend to exhibit 'fat tails' or empirical discreteness). Also, VaR is not a coherent risk measure in the sense of Artzner, Delbaen, Eber and Heath (1997 and 1999). Other measures, such as Conditional Value at Risk (CVaR), introduced by Uryasev and Rockafellar (2000 and 2002) and in the same sense Expected Shortfall by Acerbi and Tasche (2002), try to eliminate these deficiencies. These latest measures have been demonstrated to have great advantages for portfolio optimization purposes (Uryasev, Palmquist, and Krokhmal (2002)).

The empirical results of this paper show that the optimum portfolio obtained by the classic measures of portfolio management (duration, convexity, and dispersion) differs from the optimum portfolio that the modern management would obtain (using VaR and CVaR), establishing the existing relations among them. This test is conducted on maturity-constrained duration-matching portfolios. Also, the impact of bond structure (e.g. time to maturity, coupon) on the results is examined. Finally, it is verified that it happens in the optimization when the fixed investment horizon constraint is relaxed. As a result, the advantage of modern measures of risk (such as CVaR) becomes evident.

In order to obtain objectivity, this paper presents an empirical study on the US Treasury bonds market for two periods: the first, 1993-1997 and the second, 2000-2003. Furthermore, a portfolio set was created to obtain the optimization results for each measure using the historical method for the optimization with the VaR and CVaR.

OPTIMIZING WITH RISK MEASURES

Bond Duration (see Macaulay (1938)) is the simplest immunization model, assuming infinitesimal parallel shifts in the term structure of interest rates. The incorporation of Convexity makes substantial improvements on the Duration. It can also be shown that managers can increase profits of reduce losses with a greater convexity bond (Fischer and Weil (1971) and Bierwag, Kaufman, y Toevs (1990)).

Let $D = (D_1, D_2, ..., D_n)$ and $C = (C_1, C_2, ..., C_n)$ be vectors of durations and convexities of every asset $(1,...,n)$. Let $w = (w_1, w_2, ..., w_n)$ be the investment weights of every asset. Then, the duration of the total portfolio is $w^T D$. We aim to maximize the convexity of total portfolio (by $w^T C$) under the constraint that portfolio duration must be equal to the investment horizon m. In order to reduce the interest rate risk (considering parallel shifts) an immunized portfolio can be created solving the equation (1):

Maximize (in w) $w^T C$

Subject a:

$$w^T D = m \qquad (1)$$

$$w' \cdot \mathbf{1} = 1$$

$$w \geq 0$$

The quadratic measure M^2 and the linear dispersion $\tilde{N}$ improve the immunization of portfolios when the shifts in the term structure are not parallel. The objective of these measures is to decrease the interest rate risk and to create an immunized portfolio (Balbás, Ibáñez, Lopez, (2002)).

Let r be the yield to maturity and F_t the security cash flow in the period t. Now we can define the corresponding dispersion measure M^k being $k = 1$ for $\tilde{N}$ and $k = 2$ for M^2:

$$M^k = \frac{\sum_{t=1}^{n} \frac{F_t}{(1+r)^t} (t-m)^k}{\sum_{t=1}^{n} \frac{F_t}{(1+r)^t}} \quad (2)$$

The dispersion tries to measure the variance of cash flows around the investment horizon, (m) of a bond.

Let $M^k = (M_1^k, M_2^k, ..., M_n^k)$ be the bond dispersion vector. The portfolio dispersion is $w^T M^k$, and we intend to minimize it under the constraint that portfolio duration equals the investment horizon m. In order to reduce the interest rate risk (assuming nonparallel shifts), an immunized portfolio can be created by solving the following equation (3):

Minimize (in w) $w^T M^k$

Subject to:

$$w^T D = m \qquad (3)$$

$$w' \cdot \mathbf{1} = 1$$

$$w \geq 0$$

Optimization using modern measures of risk has the objective to obtain a portfolio that minimizes VaR or CVaR for a specific confidence level α (Pflug (2001)). Taking into account that VaR at level α is the absolute value of the worst loss not to be exceeded with a probability of at least 1-α. Meanwhile, the CVaR is the expected value of the loss in the 1-α worst cases. For that, let $\xi = (\xi_1, \xi_2, ..., \xi_n)$ be a vector of random returns associated to assets. The portfolio return is then calculated as $-Y = w'\xi$, where the objective is to minimize the VaR under the constraint that the expected return exceeds a certain level ($w'E(\xi) \geq \mu$). The optimization problem for the CVaR (the same as for VaR) is summarized on equation (4):

Minimize (in w) $CVAR_{\alpha}(-w'\xi)$

Subject a:

$$w'E(\xi) \geq \mu \qquad (4)$$

$$w' \cdot \mathbf{1} = 1$$

$$w \geq 0$$

EMPIRICAL STUDY

To bring more light to the behavior of each risk measure, an empirical study has been conducted, showing the results of portfolio optimization. The test has been centered in US Treasury bonds, default free and option free bonds with a liquid market. For this purpose, 11 bonds[1] with maturities of 1, 2, 3, 5, 7, 10 and 20 years have been chosen. They also have diverse coupon rates to analyze the effect of this variable. Creating 21 portfolios of two bonds each one, it was possible to analyze the behavior of the classic measures as it was of modern measures and to reach the conclusions on the differences between them.

Table I shows the bonds chosen for the analysis and their classic measures of risk. The first column is the number assigned to the bond, the second is maturity (in years), the third is the yield to maturity (YTM), the fourth is the coupon (as a percentage), the fifth is the duration (in years), and the last three columns correspond to the convexity, M^2 and $\tilde{N}$ respectively.

TABLE I
CHARACTERISTICS OF BONDS

Bond no.	Maturity (years)	YTM (%)	Coupon (%)	Duration (years)	Convexity	M^2	$\tilde{N}_{\vert}$
1	1	1.064%	5.250%	0.96846	1.4488	16.2591	2.0158
2	1	1.072%	7.250%	0.96392	1.4529	16.2978	2.0180
3	2	1.295%	6.500%	1.89248	4.7016	9.7507	1.5538
4	2	1.203%	12.000%	1.82940	4.5993	10.2043	1.5853
5	3	1.618%	2.000%	2.90829	9.9728	4.5023	1.0459
6	3	1.622%	4.625%	2.82316	9.6942	5.0015	1.0884
7	5	2.294%	2.625%	4.70326	25.0177	0.9015	0.1484
8	5	2.297%	5.625%	4.45058	23.4902	1.7139	0.2747
9	7	2.890%	5.750%	6.12530	43.8626	5.2826	1.0854
10	10	3.306%	3.875%	8.25620	77.9711	18.0137	2.0169
11	20	4.227%	7.125%	12.20350	196.3140	99.3028	4.1412

For the calculation of the historical yield shifts we used the data from the FED. The series are weekly, and to avoid possible distortions caused by the volatility, two series in different periods have been chosen:

- The first from January 2000 to May 2003.
- The second from October 1993 to February 1997.

Both series consist of 177 weekly observations. The chosen historical yields to maturity are taken from US Treasury bonds with maturities of 1, 2, 3, 5, 7, 10 and 20 years, according to the maturity of bonds used for the analysis, thus allowing the calculation of the real yield shifts.

For the test, we took an investment planning horizon of five years (m = 5), along the line of empirical studies on immunization. For this purpose, the duration is considered the first immunization measure. For that, we calculate the proportions of each instrument in the portfolio in order to obtain a duration of exactly five years.

Table II shows the matching duration portfolios created with pairs of bonds previously described. The first column is the portfolio number, the second is the first bond in the portfolio, the third column is the second bond in the portfolio, the fourth is the first bond percentage, and the last tree columns correspond to the convexity, M^2 and $\tilde{N}$ respectively.

TABLE II
MATCHING-DURATION PORTFOLIOS

No	1st Bond	2st Bond	% (1st)	Convexity	M^2	$\tilde{N}$
1	1	9	21.822%	34.607	7.678	1.288
2	1	10	44.681%	43.781	17.230	2.016
3	1	11	64.116%	71.373	46.058	2.778
4	2	9	21.802%	34.616	7.684	1.289
5	2	10	44.653%	43.804	17.248	2.017
6	2	11	64.090%	71.427	46.105	2.780
7	3	9	26.585%	33.452	6.470	1.210
8	3	10	51.168%	40.480	13.786	1.780
9	3	11	69.862%	62.449	36.740	2.334
10	4	9	26.195%	33.578	6.572	1.216
11	4	10	50.666%	40.797	14.057	1.798
12	4	11	69.437%	63.192	37.435	2.366
13	5	9	34.980%	32.008	5.010	1.072
14	5	10	60.887%	36.569	9.787	1.426
15	5	11	77.497%	51.905	25.835	1.742
16	6	9	34.078%	32.219	5.187	1.086
17	6	10	59.933%	37.051	10.215	1.460
18	6	11	76.794%	53.002	26.885	1.797
19	7	9	79.133%	28.950	1.816	0.344
20	7	10	91.648%	29.440	2.331	0.304
21	7	11	96.044%	31.795	4.795	0.306
22	8	9	67.193%	30.174	2.885	0.541
23	8	10	85.563%	31.356	4.067	0.526
24	8	11	92.913%	35.738	8.630	0.549

THE RESULTS

If the shifts on term structure of interest rates are parallel, after having fixed the portfolio duration equal to the investment horizon, the optimum portfolio using the duration-convexity method will be the one with the highest convexity. In this sense, *Portfolio 6 has* a higher convexity, and the *Portfolio 3* follows it. On the other hand, the optimization using dispersion measures will recommend the portfolio that minimizes the dispersion measures. Of this form, *Portfolio 19* and Portfolio *20* would be the best ones since *Portfolio 19* exhibits a lower M^2, and *Portfolio 20* has a lower $\tilde{N}$.

For the analysis of the behavior of both the VaR and CVaR in the portfolio optimization we used the historical method, calculating the price of each bond as a function of the historical yield shifts. These shifts were gathered from the seven series of past yields to maturity for each period (2000-2003 and 1993-1997), considering the real shifts on term structure of interest rate. Following, we proceed to calculate the percentage variations of bond prices, finally calculating the returns of each of the 24 portfolios, whose durations are equal to five years.

Table III (for 1993-1997 period) and Table IV (for 2000-2003 period) summarize the VaR and CVaR calculations using different confidence levels (90%, 95% and 99%) for the first 10 portfolios (which show the lowest risk).

TABLE III
VAR AND CVAR OF MAIN PORTFOLIOS 1993-1997

No	1st Bond	2st Bond	% (1st)	VaR 90%	VaR 95%	VaR 99%	CVaR 90%	CVaR 95%	CVaR 99%
1	1	9	21.8%	3.471%	5.039%	8.471%	5.268%	6.423%	9.176%
2	1	10	44.7%	3.699%	4.486%	**6.099%**	**4.657%**	**5.394%**	**6.481%**
3	1	11	64.1%	3.026%	**4.352%**	6.156%	4.742%	5.730%	7.362%
4	2	9	21.8%	3.539%	5.564%	8.446%	5.458%	6.653%	9.182%
5	2	10	44.7%	3.758%	4.807%	6.725%	5.030%	5.825%	6.787%
6	2	11	64.1%	**3.021%**	4.547%	7.054%	4.994%	6.219%	7.800%
7	3	9	26.6%	3.431%	5.477%	9.060%	5.432%	6.703%	9.153%
8	3	10	51.2%	3.561%	4.376%	7.094%	5.080%	6.195%	7.186%
9	3	11	69.9%	3.215%	4.431%	7.779%	5.104%	6.398%	8.652%
10	4	9	26.2%	3.607%	5.525%	9.305%	5.525%	6.898%	9.430%

TABLE IV
VAR AND CVAR OF MAIN PORTFOLIOS 2000-2003

No	1st Bond	2st Bond	% (1st)	VaR 90%	VaR 95%	VaR 99%	CVaR 90%	CVaR 95%	CVaR 99%
1	1	9	21.8%	**1.313%**	2.972%	18.367%	6.401%	10.668%	20.768%
2	1	10	44.7%	1.950%	5.766%	**13.125%**	**6.319%**	10.335%	**15.008%**
3	1	11	64.1%	1.851%	3.778%	15.185%	6.429%	**10.197%**	15.675%
4	2	9	21.8%	1.671%	2.953%	21.875%	6.482%	10.935%	22.298%
5	2	10	44.7%	2.022%	**2.891%**	16.467%	6.455%	10.652%	19.441%
6	2	11	64.1%	1.475%	4.049%	16.211%	6.562%	10.640%	19.254%
7	3	9	26.6%	1.666%	3.602%	21.171%	6.503%	10.891%	21.613%
8	3	10	51.2%	1.446%	3.546%	14.399%	6.650%	10.800%	18.636%
9	3	11	69.9%	1.531%	4.008%	18.210%	6.749%	10.793%	20.265%
10	4	9	26.2%	1.869%	3.785%	21.145%	6.669%	10.911%	21.686%

In the VaR optimization there is a significant discrepancy among confidence levels:

- 2000-2003: *Portfolio 1* to 90%, *Portfolio 5* to 95%, and *Portfolio 2* to 99%.
- 1993-1997: *Portfolio 6* to 90%, *Portfolio 3* to 95%, and *Portfolio 2* to 99%.

This discrepancy is explained because the return distributions show empirical discreteness which cause different results.

Finally, when the election of the optimal portfolio is made using CVaR, the following results are obtained:

- 2000-2003: *Portfolio 2* to 90% and 99%, and *Portfolio 3* to 95%.
- 1993-1997: *Portfolio 2* to 90%, 95% and to 99%.

It is worth noting that *Portfolio 2* and *Portfolio 3* statistics (specifically, return distributions) are very similar on the 2000-2003 period. For that reason, the optimal portfolio can change when the confidence interval varies.

With respect to the bond structure, it is necessary to remember that the first eight bonds were selected in pairs with the same time to maturity but with different coupon rates. For example *Bond 3* and *Bond 4* have the same maturity but the coupon rates are different (out of the 24 portfolios, half of them show differences in the coupon rates). Portfolios containing lower coupon bonds show lower CVaR than equivalent portfolios with higher coupon bonds. On the other hand, when the duration restriction is imposed, portfolios composed of higher coupon rate bonds show a higher convexity but also a higher CVaR than their equivalents formed by low coupon bonds.

CONCLUSIONS

The availability and increasing popularity of new risk measures to manage interest rates risk in fixed income portfolios emphasize the importance of comparing them to classic interest risk measures in order to validate their advantages.

It is important to note that the practice to maximize convexity not always implies a CVaR optimization, since finding the most convex portfolio is not equivalent to finding then minimum CVaR portfolio.

Regarding the instrument structure it is possible to conclude that portfolios composed of bonds with lower coupons have a lower CVaR than comparable portfolios containing bonds with higher coupons.

There is an absence of relationships between those portfolios optimized by classic measures and those optimized by modern measures. Modern measures collect the historical interest rates behavior; contrastingly, dispersion measures assume that the variations in interest rates are not parallel, even though parallel movements can be possible in real life.

Bond portfolios composed of lower yield volatility instruments are most desirable when modern measures are used to minimize risk. In addition, when the optimization is made using VaR, the resulting portfolios differ significantly among them, according to chosen confidence level; whereas when using CVaR results show much higher stability.

Finally, an interesting issue for further research would be to incorporate fixed income derivatives and to measure the performance and effectiveness of both types of strategies, incorporating the application of rebalancing techniques.

NOTES

[1]The data and characteristics of bonds have been obtained from broker Bondpage.com, which is a service of Cambridge Group Investments, Ltd., a Society of Stock market. Cambridge Group Investments is a member of the National Association of Securities Dealers (NASD), Securities Investor Protection Corporation (SIPC) and Municipal Securities Rulemaking Board (MSRB).

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